Uncountable groups with some Normality Conditions on some Large Subgroups

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October 8^{th}, 2024

Motivations

- Let G be a group;
- Let Σ be a subset of $\mathcal{L}(G)$;
- Let $\mathcal{P}$ be a theoretical property of subgroups.

If for each $H \in \Sigma$, H satisfies $\mathcal{P}$...
what about G?

From Small to Large

Definition 1

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- *Countable recognizability* was introduced by R. Baer.

Baer, R.: Abzählbar erkennbare gruppentheoretische Eigenschaften. Math. Z. 79, 344–363 (1962).

Theorem 1.1

The class of nilpotent groups is countably recognizable.

From Large to Small

Definition 2

A group G is said to have finite rank if there is a positive integer r such that for every

$$H \leq_{f.g.} G, \exists x_1, \ldots, x_r : H = \langle x_1, \ldots, x_r \rangle.$$

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M. De Falco, F. de Giovanni, C. Musella. *Groups with normality conditions for subgroups of infinite rank*. Publ. Mat. 58 (2) 331 - 340, 2014.

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Let H be a proper subgroup of an uncountable group G of cardinality $\aleph$.

- H will be said *large* if $|H| = \aleph$.
- H will be said *small* if $|H| < \aleph$.

Jònsson groups

The main obstacle in the study of groups of large cardinality is the existence of (uncountable) groups G all of whose proper subgroups are small.

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Such groups are the so-called (uncountable) *Jònsson groups*!

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We have to consider groups with no-Jònsson normal subgroups.

Abelian groups of cardinality $\aleph$

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Theorem 2.1 (M. De Falco - F. de Giovanni - H. Heineken - C. Musella, 2016)

Let A be an uncountable abelian group of cardinality $\aleph$. Then A contains a subgroup of the form

$$B = \underset{i \in I}{Dr} B_i,$$

where the set I has cardinality $\aleph$ and either all B_i are infinite cyclic or they all have the same prime order.

$C^1_{\aleph}$-groups

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- The commutator subgroup of a $C^1_{\aleph}$-group either is divisible or has prime exponent and it is always **small**.

A. Ehrenfeucht – V. Faber: "*Do infinite nilpotent groups always have equipotent abelian subgroups?*", Kon. Nederl. Akad. Wet. A75 (1972), 202–209.

Normality and Subnormality of Large Subgroups

Theorem 3.1 (Roseblade)

Let G be a group and k a positive integer. If every $H \leq G$ is subnormal of defect at most k, then G is nilpotent of class at most $f(k)$.

Normality and Subnormality of Large Subgroups

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Theorem (M. J. Evans and Y. Kim, 2004)

Let G be a locally soluble group of infinite rank in which all subgroups of infinite rank are subnormal of bounded defect, then all subgroups of G are subnormal of bounded defect.

Normality and Subnormality of Large Subgroups

Theorem 3.2 (M. De Falco - F. de Giovanni - H. Heineken - C. Musella, 2016)

Let k be a positive integer, and let G be an uncountable group of cardinality $\aleph$ in which all large subgroups are subnormal of defect at most k.

(a) *If G contains an abelian large normal subgroup, then all subgroups of G are subnormal of defect at most k, and so G is nilpotent of class at most $f(k)$.*

(b) *If G contains an abelian large subgroup, then G is nilpotent and the subgroup $\gamma_{f(k)+1}(G)$ is small.*

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Theorem 3.3 (Baer)

Let G be a group. G is a Dedekind group if and only if G is either abelian or a direct product of the form $G = Q_8 \times B \times D$, where B is an elementary abelian 2-group, and D is a torsion abelian group with all elements of odd order.

Normality and Subnormality of Large Subgroups

Normality and Subnormality of Large Subgroups

Corollary 3.4

Let G be an uncountable group of cardinality $\aleph$ which has no Jònsson large subgroups. If all large subgroups of G are normal and G contains an abelian large subgroup, then G is a Dedekind group.

Normality and Subnormality of Large Subgroups

Theorem 3.5 (M. De Falco - F. de Giovanni- C. Musella)

Let G be an uncountable group of cardinality $\aleph$ which has no Jònsson large subgroups. Then all large subgroups of G are normal if and only if one of the following conditions holds:

(a) G is a Dedekind group;

(b) G is a $C^1_{\aleph}$-group;

(c) $G = P \times Q$, where P is a nilpotent p-group of cardinality $\aleph$ for some prime p, Q is a periodic Dedekind $p^{'}$-group, and

- *$p > 2$, P is a $C^1_{\aleph}$-group and Q is nonabelian,*
- *$p = 2$ and P contains a nonabelian $C^1_{\aleph}$-subgroup N of finite index and such that $P/N^{'}$ is a Dedekind nonabelian group.*

Normality in Uncountable Groups

Corollary 3.6

Let G be an uncountable group of cardinality $\aleph$ which has no Jònsson large subgroups. If all large subgroups of G are normal, then G contains a $C^1_{\aleph}$-subgroup N such that G/N is finite.

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M. De Falco, F. de Giovanni, C. Musella, *Groups with normality conditions for uncountable subgroups*, J. Aust. Math. Soc. 111 (2021), 268–277.

Nearly Normality of Large Subgroups

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Theorem 3.7 (B. H. Neumann)

Let G be a group. For all $H \leq G$, H is nearly normal if and only if $|G'| < \infty$.

B. H. Neumann, *Groups with finite classes of conjugate subgroups*, Math. Z. 63 (1955), 76–96.

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Theorem (M. De Falco - F. de Giovanni - C. Musella)

Let G be an infinite rank locally soluble group in which every subgroup of infinite rank is nearly normal. Then, $|G^{'}| < \infty$.

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Theorem 3.8 (M. De Falco - F. de Giovanni - C. Musella)

Let G be an uncountable group of cardinality $\aleph$ in which every large subgroup is nearly normal. If G contains a large FC-subgroup, then $|G^{'}| < \infty$.

Nearly Normality of Large Subgroups

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Theorem 3.9 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$. If there exists a positive integer k such that for every large subgroup X of G, $|X^G : X| \leq k$, then for every large subgroup X of G, the index $|G' : X'| \leq f(k)$.

Nearly Normality of Large Subgroups

Theorem 3.10 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ which has no Jònsson large subgroups. If there exists a positive integer k such that for every large subgroup X of G, $|X^G : X| \leq k$, then

(i) $G^{'}$ is small,

(ii) $\gamma_3(G) = [G^{'}, G]$ is finite.

If $G^{'}$ is infinite, there exists a finite subgroup N containing $\gamma_3(G)$ such that either $G^{'}/N$ has prime exponent p or G/N is a $C^1_{\aleph}$-group with divisible commutator subgroup.

Nearly Normality of Large Subgroups

Corollary 3.11

Let G be an uncountable group of cardinality $\aleph$ which has no Jònsson large subgroups. If there exists a positive integer k such that for every large subgroup X of G, $|X^G : X| \leq k$ and G is torsion-free, than G is a $C^1_{\aleph}$-group.

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$C^1_\aleph$-groups

$C^1_\aleph$-groups replace abelian groups when we restrict our attention only on large subgroups.

Nearly Normality of Large Subgroups

Theorem 3.12 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ which has no Jònsson large subgroups. Then there exists a positive integer k such that $|X^G : X| \leq k$ for every large subgroup X of G if and only if there exists a positive integer h and a family $\{K_j | j \in J\}$ such that

- *K_j is a $C^1_\aleph$-group, for each $j \in J$;*
- *$|G' : K_j'| \leq h$, for each $j \in J$;*
- *X' contains some K_j', for every large subgroup X of G.*

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- *K_j is a $C^1_\aleph$-group, for each $j \in J$;*
- *$|G^{'} : K^{'}_j| \leq h$, for each $j \in J$;*
- *$X^{'}$ contains some $K^{'}_j$, for every large subgroup X of G.*

$\Rightarrow$ The commutator subgroups of large subgroups are constrained between a fixed $K^{'}_j$ and $G^{'}$.

Further normality conditions for Large Subgroups

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Theorem 3.13 (M. J. Tomkinson)

Let G be an FC-group. Let α be an infinite cardinal. If $|U^G : U| < \alpha$ for each subgroup U of G, then $|G'| < \alpha$.

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Theorem 3.14 (M. De Falco - C. Musella - GS)

Let G be an uncountable FC-group of cardinality $\aleph$. Let α be an infinite cardinal. If $|U^G : U| < \alpha$ for each large subgroup U of G, then $|G^{'}| < \alpha$.

Almost Normality of Large Subgroups

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Theorem 3.15 (B. H. Neumann)

Let G be a group. $|G/Z(G)| < \infty$ if and only if for every $H \leq G$, H is almost normal.

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Theorem 3.16 (I. Schur)

Let G be a group. If $|G/Z(G)| < \infty$, then $|G'| < \infty$.

Almost Normality of Large Subgroups

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Theorem (M. De Falco - F. de Giovanni - C. Musella)

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Theorem 3.17 (M. De Falco - F. de Giovanni - C. Musella)

Let G be an uncountable group of cardinality $\aleph$ in which all large subgroups are almost normal. If G contains a large abelian subgroup, then $|G/Z(G)| < \infty$.

Almost Normality and Normalizers

Almost Normality and Normalizers

Theorem 3.18 (Y. D. Polovickĭ)

Let G be a group. $|G/Z(G)| < \infty$ if and only if G has finitely many normalizers of (abelian) subgroups.

Y.D. Polovickĭ *Groups with finite classes of conjugate infinite abelian subgroups*, Soviet Math. (Izv. VUZ) 24 (1980) 52–59.

Finitely many Normalizers of Large Subgroups

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Theorem 3.19 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ having no large simple sections. If G contains only finitely many normalizers of large subgroups, then

$$G' \text{ is small.}$$

Finitely many Normalizers of Large Subgroups

Theorem 3.19 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ having no large simple sections. If G contains only finitely many normalizers of large subgroups, then

G' *is small.*

Theorem 3.20 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ having no large simple sections such that G contains only finitely many normalizers of large subgroups. If G contains a large abelian subgroup, then $|G/Z(G)| < \infty$. In particular,

G' *is finite.*

Almost Normality and Normalizers for Large Subgroups

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Almost Normality and Normalizers for Large Subgroups

- Almost normality and (finite) number of normalizers are related also for large subgroups:

Theorem 3.21 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ having no large simple sections. Then the following conditions are equivalent:

(i) G contains only finitely many normalizers of large subgroups.

(ii) All large subgroups are boundedly almost normal.

Finitely many Commutator Subgroups

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Theorem 3.22 (F. de Giovanni - D. J. S. Robinson)

Let G be a group with finitely many commutator subgroups, then $|G^{'}| < \infty$.

F. de Giovanni, D.J.S. Robinson, *Groups with finitely many derived subgroups*, J. London Math. Soc. 71 (2005), 658–668.

Finitely many Commutator Subgroups

Theorem 3.23 (M. De Falco, F. de Giovanni, C. Musella, 2020)

Let G be a locally graded uncountable group of cardinality $\aleph$. If G has finitely many commutator subgroups and G contains a large abelian subgroup, $|G^{'}| < \infty$.

Finitely many Commutator Subgroups

Theorem 3.23 (M. De Falco, F. de Giovanni, C. Musella, 2020)

Let G be a locally graded uncountable group of cardinality $\aleph$. If G has finitely many commutator subgroups and G contains a large abelian subgroup, $|G^{'}| < \infty$.

M. De Falco, F. de Giovanni and C. Musella, *A note on commutator subgroups in groups of large cardinality*, Monatsh. Math. 191 (2020), 249–256.

Finitely many Commutator Subgroups of Large Subgroups

Theorem 3.24 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ having no large simple sections.

(1) If G has finitely many commutator subgroups of large subgroups, then G is finite-by-$C^1_\aleph$.

(2) If G is finite-by-$C^1_\aleph$, then all large subgroups are boundedly nearly normal.

Finitely many Commutator Subgroups of Large Subgroups

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(2) If G is finite-by-$C^1_\aleph$, then all large subgroups are boundedly nearly normal.

- G' is small.

$C^1_\aleph$-groups

$C^1_\aleph$-groups play a symmetrical role with respect abelian groups when we focus only on large subgroups!

Finitely many Commutator Subgroups of Large Subgroups

Theorem (I. Shur)

Let G be a group in which there are finitely many normalizers. Then, in G there are finitely many commutator subgroups.

Finitely many Commutator Subgroups of Large Subgroups

Theorem 3.25 (M. De Falco - C. Musella - GS)

Let G be an uncountable group of cardinality $\aleph$ having no large simple sections. If G contains only finitely many normalizers of large subgroups

$$\{N_G(X_1), ..., N_G(X_n)\},$$

then G contains only finitely many commutator subgroups of large subgroups

$$\{Y_1^{'}, ..., Y_m^{'}\}.$$

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M. De Falco, C. Musella, G. Sabatino, *Uncountable groups with small commutator subgroup*, Journal of Algebra and Its Applications (2024).

M. De Falco, C. Musella, G. Sabatino, *Uncountable groups with finitely many normalizers of large subgroups*, Arch. Math. 122, 369–376 (2024).

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- Normality conditions on (a set of) subgroups of a group G have a high impact on the cardinality of the commutator subgroup G' of G and vice versa.

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- Normality conditions on (a set of) subgroups of a group G have a high impact on the cardinality of the commutator subgroup G' of G and vice versa.
- The behaviour of the large subgroups of an uncountable group has a strong influence on the structure of the whole group.

THANK YOU

FOR

YOUR ATTENTION!