



Carlo Casolo (1958 – 2020)



On March 28th Carlo Casolo passed away and his death left us all (friends, colleagues, students) in total disbelief. Still now, after months from that day, it is hard to fully understand the significance of his lost and the profound impact that Carlo's absence is going to have on the lives of all the persons who, because of personal friendship or mathematical reasons, were close to him. Anyone who

wants to get an idea of what Carlo meant for people who had the chance to meet him, should have a look at the webpage <https://sites.google.com/unifi.it/algebra-al-dini/algebra-al-dini/carlo-casolo> where anyone who wished, left some thoughts as a farewell. It is particularly touching to read what his students, old and recent, have written.

As a mathematician Carlo made significant contributions in several areas of group theory, but there are two particular subjects that deserve to be mentioned. The first one is, without doubt, that of *subnormal subgroups*, a theme that he started investigating in his undergraduate thesis and that became one of the main topics of interest for him. In particular we must mention his work on groups with all subgroups subnormal (the so called \mathcal{N}_1 -groups) culminating in the papers [C2] and [C3] where he describes the structure of \mathcal{N}_1 -groups

and proves that \mathcal{N}_1 -groups are Fitting groups, thus solving a long-standing open problem. He was a leading expert in this area and his knowledge of the subject transpires from reading the impressive paper [C4] where Carlo was able to clearly explain a wealth of difficult techniques involved in this area guiding the reader to the proofs of some of the deepest results about \mathcal{N}_1 -groups.

Another series of amazing results can be found in his papers on conjugacy classes and related topics. Some of them, like [C1] (the proof of the ρ - σ conjecture for perfect groups), [CD] (the proof of one of the very few known connections between character's degree and conjugacy class sizes), [C5] (an extension, obtained without the use of CFSG, to all finite groups of a result known for solvable groups) or [ACDKP] (a surprising result about the character prime graph of solvable group), are examples of Carlo's ingenuity and insight. He made many other contributions in different areas of group theory but, rather than discussing them, I prefer just to point out that, contrary to what has become more and more common, his work spaces from finite to infinite groups, a fact that rarely happens nowadays. In all his papers we may notice the ingenuity and elegance of the arguments, as well as the high standards Carlo set for himself. An area in which Carlo was very active in his latest years, was the one of *mathematics and literature*. He was a person of culture, loved every form of art and was particularly fond of literature and music. He had a thorough knowledge of both of them and his personal reflections on these subjects were always original and enlightening.

Besides being an active researcher Carlo devoted an incredible amount of energies to teaching and mentoring. In the above mentioned webpage one can read how our students were impressed by Carlo's way of teaching. His lectures were always clear and he was able to pass over to students the love he had for mathematics. Moreover, in spite of the large burden of bureaucracy he was charged with, he always had time to talk to students who often showed up in his office seeking help or advice. It is therefore no surprise to see how much he was loved and respected by his students even though passing his exams was all but easy. Another sign of Carlo's commitment in teaching is the number of lecture notes he had written over the year, as a support for the courses he taught. What started as a rough set of notes, grew up into some impressive textbooks that, by the way, were noticed by some editorial houses. Indeed Carlo received several offers to turn those notes into books, but all these offers were turned down since, as he was used to say, that material was meant

to be used freely by everyone. These notes are (and will be) available on his webpage. They consist of a course in basic algebra, one in commutative algebra, one in graph theory, one in group theory one on geometric group theory and a course in elementary number theory. All these notes are examples of clarity and reflect some of Carlo's ideas on teaching (and learning) mathematics. In all of them exercises and examples abound, as Carlo was firmly convinced that mathematics cannot be learned without some *bare hands* work. Carlo supervised a large number of PhD students and the subjects of their thesis cover a wide area of group theory. Carlo's insight was so deep that he has been able to guide his students through themes quite far from his *comfortable zone*. Many of his students are now well-known members of our group theory community. Doing research and teaching do not exhaust the duties of an academic and Carlo made not exception. Although he did not like this side of his job, he was involved in many other duties related to the administration of our institution. He served as a chairman of our department and has been member of innumerable boards, from the ones in charge of the organization of teaching to the ones who had to decide about sharing the funds or appoint new positions. Although he did not like spending time in endless meetings, he considered this as part of his duties and always carried out these tasks with the maximum care. He was indeed a well respected member of the academic community and his opinion was always taken in great consideration. His equilibrium, integrity and honesty made him the right person to be appointed when some delicate matter had to be managed.

I met Carlo in 1986 while I was still an undergraduate student, so our friendships lasted for more than three decades, becoming more and more solid in the last twenty years during which we shared the same office at the math department in Firenze. Meeting Carlo every day was so *natural* for me that I can not believe I will never see him again, sitting at his desk in front of me. The sadness I feel can not be expressed but, on the other hand, I know that his heritage will not be lost and every person who has been in contact with him, will pass over what Carlo has so generously given.

No one is finally dead until the ripples they cause in the world die away, until the clock wound up winds down, until the wine she made has finished its ferment, until the crop they planted is harvested. The span of someone's life is only the core of their actual existence – Terry Pratchett

Orazio Puglisi

References

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