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A History of Group Theory through the Lives of Group Theorists

Sophus Lie - Part 1

We continue here our attempt of a systematic historical account of Group Theory inspected by means of the lives and the works of its main actors. The aim is to bring the interested reader through original correspondences, published and unpublished works, historical perspectives, diatribes and friendships.

This issue contains the translation of a memory of Sophus Lie written by Ludwig Sylow. It was published in the 1899 issue of *Archiv for Mathematik of Naturvidenskab* soon after Lie's death.

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Sophus Lie

Marius Sophus Lie was born on December 17th, 1842 in the village Nordfjordeid. His father was a priest that moved later to the town Moss, whence the young Sophus was sent to Christiana to study at the private school *Nissens Latin- og Realsskole*. As a young lad he was evidently, without comparison, the best mathematics student in his class. However, while Abel neglected his other school subjects in order to focus on higher mathematics, and at once started to work on a problem which later would be solved by him, Lie was good in all subjects and had no special calling for any particular discipline.

He graduated with distinction in 1859 with outstanding marks in all subjects apart from Norwegian writing. He started with uncertainty as to what to study afterwards. He was not aware of his exceptional mathematical gifts and quite on the contrary he later said that the road to Mathematics was long and difficult for him. After some wavering he opted to study to obtain teaching qualifications in mathematics and natural sciences, which he passed with excellent marks in 1865. In 1862 he had followed a short course on the theory of substitutions through which he became familiar with the concept of a Galois group that had influence on his later development. But what gave him the early stimulation and made him realise *that there lurked a mathematician in him*, to quote his own words, was however the study of modern geometry, a subject which, before Lie, was given little attention at our university, but that had profound impact on Lie. Lie was in fact a natural geometer; he worked essentially with geometrical settings and geometrical images, even when he was broadly working within analysis. His first work is in 1869 and involves a representation of planar geometrical objects, whereby theorems in planar geometry are transformed into stereometric theorems, where one replaces a point in the real or complex plane by a real straight line in the space. As his exceptional gifts were becoming clear to himself and others, he applied for and obtained a stipend to go abroad, and spent the autumn 1860 in Germany, followed by a period in Paris. This trip was important for the path that Lie would take later. Certainly he brought with him from home his core ideas, wherein he had already started his research on transformation groups and differential equations, but the contact with many outstanding mathematicians influenced to a large degree how these core ideas developed. He spent a short time under the supervision of Prof. A. Cleb-

sch in Götttingen, and made acquaintance with the brilliant young mathematicians F. Klein, C. Jordan and G. Darboux and probably many others through this group. In particular, he formed a friendship with Klein, who was Plücker's student and publisher of his latest papers. In collaboration with Klein he studied curves and surfaces that are invariant under a group of linear transformations and presented some of his results to the Paris academy. The outbreak of the Franco-German war at this moment put an end to their stay together in Paris, but for many years there was a lively exchange of ideas between them. Lie, a strong walker, wanted to travel through France on foot but didn't get further than Fontainebleau, where his wander was treated as suspicious and he was arrested and his papers only made the situation worse. In vain he appealed to French mathematicians; the blond giant looking too much like a German. At last Darboux was found, and could easily convince the authorities that the suspicious notes were mathematical formulas and figures, and that the German name belonged to a known mathematician. He was thus finally released and continued his trip through Switzerland to the north of Italy and returned home at the end of the year.

Soon after his return Lie received one of the university research scholarships. He continued his geometric studies, that had now developed more and more into the study of geometric transformations. He had come up with a peculiar transformation that transforms straight lines of the space into spherical surfaces, the foundation for spherical geometry, and this had led to a remarkable result, where he could determine the asymptotic curves of the fourth degree Kummer surface with 16 knot points, and show that they are algebraic; this was presented to the Berlin Academy in 1870. This transformation now became a basis for a thesis that he submitted and defended to obtain a PhD in 1871. In this thesis, and in particular in the subsequent work, Lie used also his geometric transformations to study differential equations, that now became his central theme. Also in 1871 he published jointly with Klein a work in Mathematische Annalen where transformations are again used to study differential equations and the notion of a group as well as a number of group theoretical terms are used for the transformations; now these were also done infinitesimally. Lie was now also working within the field of continuous transformations.

During the winter 1871-1872, Lie applied for a vacant professorship in Lund. But the references about his scientific work that came

from many leading mathematicians of the time led to a special professorship being created for him at our own university. The appointment was in July 1872. He now worked as a professor in Christiania until the year 1886; from now on he could devote his time on his research and the result was an exceedingly rich product of original work.

The problem of integrating partial differential equations of the first order became the subject of a series of papers, and he succeeded in simplifying the older methods, due to mathematicians such as Cauchy and Jacobi. In this context, he met his friend and later colleague Prof. A. Mayer in Leipzig, who at the same time and independently from Lie, studied the same questions; the two researchers had come to exactly the same results, and it led to an unusual collaboration between them, as one took up and worked out the ideas of the other.

Soon after, especially since the year 1876, when he, together with Professors Worm-Muller and G.O. Sars founded this journal, his publications focused around his own studies, namely the theory of continuous transformation groups. He expanded the original concept to include contact transformations, thereby creating a new and wide ranging theory of importance for many, almost all the areas of mathematics. Others contributed to the theory by using it to integrate certain partial differential equations of the second order, and even classes of these, later also general differential equations that are preserved under a known transformation group, something that had already been touched on in one of his earlier joint work with Klein.

Nor did he forget the actual geometry; he did extensive work on minimal surfaces, surfaces with constant curvature, translation surfaces, and in each of these areas, he provided a plethora of new results.

In 1886, Lie was offered a professorship in Leipzig. As his professorship in Christiania was personal, he was allowed to take a sabbatical while taking up the new position and was free to return back to his old post, something that probably had been his plan from the beginning. The arrival in Leipzig marked the beginning of the last episode of Lie's life that differed much from what his life had been like previously. In Germany, the conditions for a leading scientist are quite different than here; not only is the literature and other help resources richer, the interests of the general public larger, but one can count on being always surrounded by a circle of students, that

study science as something of value in itself. This did not happen immediately to Lie, because in Germany the students tend to follow their lecturer as he moves from one university to another, and Lie's superior in the post was just one of Germany's greatest mathematicians, namely his old friend Klein. But soon his growing reputation brought a crowd of students around him; and these came not only from Germany, as these were joined by students from France and America, and then of course young men with excellent gifts. Lie continued publishing high quality papers, that now appeared usually in the Saxon scientific *Berichte*; but in addition he gathered his work on transformation theory as a collection in three volumes *Theorie der Transformationsgruppen*, that was published by Teubners Forlag in Leipzig in the period 1888–1893. Here he was assisted by a younger German scientist, Prof. F. Engel, primarily for language and style; but of course his assistance was not restricted to this. Prof. Engel had already met Lie in 1884 when, upon suggestion of the Leipzig mathematicians Mayer and Klein, he travelled to Christiania to spend some time there to absorb Lie's theories and to assist him in the preparation of a work which Lie had already decided to publish. Additionally, Lie's lectures on differential equations and his lectures on continuous groups, both produced by his student Dr. G. Scheffers, the first closely monitored by Lie and both with his strong guidance, were published in 1891 and 1893, respectively. But Lie had more ambitious plans. He wanted to publish a work, that would be a self contained account of the theory of differential invariants that would then lead to the theory of infinite continuous groups, and in addition there would be an application of these theories for the integration of differential equations. Another work should include an elaborate presentation of his studies in geometry. This has also resulted in a first volume under the title *Geometrie der Berührungstransformationen dargestellt von Sophus Lie und Georg Scheffers* (Leipzig, 1896). The introduction says that it contains a representation of Lie's geometric work in the years 1869-1872. So the work is still far from being complete.

Unfortunately, he would not live to see these plans go through. Despite strong constitution, the intense work had taken its toll. Once he was suffering so badly from lack of sleep that he needed to take a break to seek medical treatment. He did regain some energy but still did not feel that well; the climate of Leipzig was not compatible with his physical condition. The desire to return home became stronger and when this became known, the parliament generously augmented

his salary to 10.000 kronas, so that the return would not restrain him financially. He got released from his post in Leipzig and took up his position at the University of Christiania from the second semester of 1898. But his health faltered upon arrival. It had been his intention to give two series of lectures on differential equations, one for more advanced students and one for beginners. For the former there were three American students, that had their lessons at his home, as soon he was not able to leave home. The second series never got started. The condition deteriorated slowly; he was only short time bedridden and passed away without pain on February 18th, 1899. The cause of death was anaemia. He was buried at the expense of the university with a gracious ceremony; the king and the parliament sent their representatives and the general attendance was extraordinary. His wife Anna Sophie, born Birch and related to Nils Henrik Abel on the mother side, survives him with two daughters and one son.

Lie's greatest legacy is for having created the theory of continuous transformation groups. He has himself been involved with its application in Geometry, in the integration of differential equations, in the half philosophical chapter on the elementary foundation of geometry, and in the theory of complex numbers. Presumably, the future will show that this is not all. He was widely regarded as one of the very greatest mathematicians of the current era, and there has been no lack of academic honours. I only mention that he was a member or correspondent of the *Royal Society of London*, *Académie des Sciences* in Paris, *Accademia dei Lincei* in Rome and the academy in Petersburg. But his greatest honour lies in the general influence that his theories have had, as a number of mathematicians, including some famous men, have gone further along Lie's new paths, and also some of his results are found in textbooks and used in higher studies.

Lie's death was a major loss for science, if one only looks at the planned projects that were not completed; but he left us relatively young, giving hope for new discoveries. For our science and university the loss is irreparable, so much more for the men, who are actually called upon to continue his research into the longer future, his own students, almost exclusively foreigners.

L. Sylow