A Maximal Subgroup $2^{4+6}{:}(A_5 \times 3)$ of $G_2(4)$ Treated as a Non-Split Extension $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$

Abraham Love Prins

Abstract

The maximal subgroup $2^{4+6}{:}(A_5 \times 3)$ of the Chevalley group $G_2(4)$ is isomorphic to a non-split extension group of the shape $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$. In this paper, the ordinary character table of $2^{4+6}{:}(A_5 \times 3)$ will be re-calculated using the technique of Fischer-Clifford matrices and where $2^{4+6}{:}(A_5 \times 3)$ will be treated as the non-split extension $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$. The author uses some relevant techniques to identify and compute the type of characters (ordinary or projective) of the inertia factor groups H_i of $\overline{G}$ on $\mathrm{Irr}(2^6)$, which are required in the construction of the character table of $\overline{G}$ via Fischer-Clifford theory. Also, this is a very good example to demonstrate how to apply Fischer-Clifford theory to a non-split extension group $N^{\cdot}G$, where not every irreducible character of N can be extended to its inertia group $\overline{H}_i$ in $N^{\cdot}G$.

1 Introduction

The Chevalley-Dickson simple group $G_2(4)$ of Lie type G_2 over the Galois field $GF(4)$ and of order $251596800 = 2^{12} \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$ has exactly eight conjugacy classes of maximal subgroups [4]. One

of these classes of maximal subgroups of $G_2(4)$ is represented by a 2-local subgroup of the form $2^{4+6}{:}(A_5 \times 3)$, where 2^{4+6} is a special 2-group of order 1024 with center $Z(2^{4+6}) = 2^4$. We denote this maximal subgroup by $\overline{G}$ which is also a parabolic subgroup of $G_2(4)$. The group $\overline{G}$ has index 1365 in $G_2(4)$ (see [4]).

In the ATLAS [4] of finite groups we found that $2^{4+6}{:}(A_5 \times 3)$ is the normalizer $N_{G_2(4)}((2A)^4))$ of an elementary abelian 2-group $(2A)^4$ of order 16 in $G_2(4)$, where the generators of $(2A)^4$ are 4 commuting involutions found in the class of involutions $2A$ of $G_2(4)$. The group $G_2(4)$ has a permutation representation of degree 416 [24] and we can readily construct $2^{4+6}{:}(A_5 \times 3)$ in $G_2(4)$ using the computer algebra systems MAGMA [3] or GAP [6]. The special 2-group 2^{4+6} is isomorphic to $2^4.2^6$, where both of 2^4 and 2^6 are normal subgroups in $\overline{G}$. Hence $\overline{G}$ can be constructed as a non-split extension group of the form $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$ or $\overline{G}_1 = 2^{4\cdot}(2^6{:}(A_5 \times 3))$ (see [15]).

The method of coset-analysis (see [12] and [13]) is used to compute the conjugacy classes of an extension group $\overline{G} = N.G$, where N is abelian and the conjugacy classes of $\overline{G}$ lie over the conjugacy classes of G, i.e., the classes of G are lifted to the classes of $\overline{G}$. For $g \in G$, $\overline{g}$ is a lifting of g in $\overline{G}$ under the natural homomorphism

$$\overline{G} \longrightarrow G.$$

A coset $N\overline{g}$ is considered for each class representative g in G, where $N\overline{g}$ splits into k orbits under the action of N by conjugation. Then under the action of $\{\overline{h} : h \in C_G(g)\}$, f_j of the k orbits fuse together. Hence the order of the centralizer $C_{\overline{G}}(x)$ for each element $x \in \overline{G}$ in a conjugacy class $[x]_{\overline{G}}$ is given by

$$|C_{\overline{G}}(x)| = \frac{k|C_G(g)|}{f_j}.$$

If $\overline{G} = N.G$ is a finite extension group and N is a normal p-subgroup of $\overline{G}$, then the method of Fischer-Clifford matrices [5] involves the construction of a non-singular square matrix $M(g)$, called a Fischer-Clifford matrix, for each conjugacy class $[g]$ of $\overline{G}/N \simeq G$. The top of a column of a Fischer-Clifford matrix $M(g)$ is labeled by the centralizer order $|C_{\overline{G}}(x_i)|$, for each class representative x_i obtained from the coset $N\overline{g}$ using the technique of coset analysis. The rows of the matrix $M(g)$ are labeled to the left by the centralizer or-

ders $|C_{H_i}(h)|$ of the so-called inertia factors H_i of $\overline{G}$, where h is contained in the class $[h]$ of H_i which fuse into the class $[g]$ of G. The inertia factor H_i is the quotient group

$$H_i \simeq \frac{\overline{H_i}}{N},$$

where $\overline{H_i}$ is the inertia group of a orbit of $\overline{G}$ on $Irr(N)$, the set of ordinary (complex) irreducible characters of N. In practice, the arithmetical properties of the Fischer-Clifford matrices (see, for example, [1] and [13]) are used to compute the entries of $M(g)$. The irreducible complex characters of $\overline{G}$ can be constructed from these Fischer-Clifford matrices $M(g)$ and the irreducible (ordinary or projective) characters of the subgroups H_i of G. A brief description of the technique of Fischer-Clifford matrices is given in Section 3. Readers are referred to [2] on a survey of Fischer-Clifford theory.

In this paper, some useful techniques are developed to identify and compute the type of characters of the inertia factor groups H_i which are required in the construction of the ordinary character table of the non-split extension group $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$. In this regard, readers are referred to the paper [1] and [15]. If one is only interested in the character table of $2^{4+6}{:}(A_5 \times 3)$, then it can be easily obtained from the GAP library or computed within GAP. But to compute the ordinary character table of $\overline{G}$ by means of Fischer-Clifford theory, brings out some interesting aspects of the group structure which are unique to $\overline{G}$, as it will be seen in the sections that will follows. Computations are carried out with the aid of the computer algebra systems MAGMA and GAP and the notation of ATLAS is mostly followed.

2 Preliminary results on projective characters

Since we are dealing with a non-split extension $\overline{G}$, the theory of projective characters will play an important role in the construction of the character table of $\overline{G}$ via Fischer-Clifford theory. In this section, we give some definitions and make some remarks on projective representations. A proof of a proposition (due to G. Robinson in [20]), which states that the number $|IrrProj(G, \alpha)|$ of irreducible projective characters of a finite group G associated with some factor set α is always

less or equal to the number $|\text{Irr}(G)|$ of the ordinary irreducible characters of G, is also given. Furthermore, J. Schmidt developed GAP codes (see [22]), which are based on this proposition, to compute the number of irreducible projective characters of a finite solvable group G associated with a certain factor set α. Interested readers are referred to [1],[9],[10],[11] and [19] for definitions on concepts and proofs of theorems concerning ordinary and projective character theory. In the following G will be understood to be a finite group.

Definition 2.1   *A function $\alpha : G \times G \rightarrow \mathbb{C}^*$ is called a factor set of G if $\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z)$ for all $x, y, z \in G$.*

The set of all equivalence classes of factor sets of G forms a finite abelian group $M(G)$, called the Schur Multiplier, where $M(G)$ is isomorphic to the second cohomology group $H^2(G, \mathbb{C}^*)$ of G.

Definition 2.2   *A mapping $P : G \rightarrow GL(n, \mathbb{C})$ is called a projective representation of G of degree n with associated factor set α if*

(i) $P(1_G) = I_n$, *and*

(ii) $P(x)P(y) = \alpha(x, y)P(xy) \; \forall x, y \in G.$

A projective character κ of G is defined as $\kappa(g) = \text{trace}(P(g))$ for all $g \in G$. We say that κ is irreducible if P is, and κ has a factor set α, where α is the factor set of P.

Let $\text{IrrProj}(G, \alpha)$ denote the set of irreducible projective characters of G associated with the factor set α. An element $x \in G$ is said to be α-regular if
$$\alpha(x, g) = \alpha(g, x)$$
for all $g \in C_G(x)$. It is well known that $g \in G$ is α-regular if and only if $\kappa(g) \neq 0$ for some $\kappa \in \text{IrrProj}(G, \alpha)$ or equivalently that g is α-irregular if and only if $\kappa(g) = 0$ for all $\kappa \in \text{IrrProj}(G, \alpha)$. The number of irreducible projective characters with factor set α equals the number of α-regular classes of a group G. Projective characters also satisfy the usual orthogonality relations and have analogues to ordinary characters.

Definition 2.3   *A group R is a representation group for G if there exists a homomorphism π from R onto G such that* (i) $A = \ker(\pi) \simeq M(G)$, *and* (ii) $A \leqslant Z(R) \cap R^{'}$.

A covering group C for G will normally be a quotient of R by a subgroup B of A. If A/B has order n we sometimes refer to the covering group as a n-fold cover of G. Projective representations of G are found in the representation group R for all the equivalence classes of factors sets in M(G). However, in a n-fold cover C of G only the n equivalence classes which C covers will be represented.

The following definition is taken from [21] and will be used in Proposition 2.5.

Definition 2.4   *Let* $S \leqslant G$, $\chi \in \mathrm{Irr}(G)$ *and* $\varphi \in \mathrm{Irr}(S)$. *The character* χ *is said to lie over* φ, *if* χ *is an irreducible constituent of* φ^G, *i.e.* $\langle \chi, \varphi^G \rangle \neq 0$. *By the Frobenius reciprocity this is equivalent to say that* φ *is a constituent of* χ_S, *i.e.* $\langle \chi_S, \varphi \rangle \neq 0$. *In this case, we also say that* φ *lies under* χ.

The following proposition (see [16] and [20]) is useful to determine the number of irreducible projective characters of a group G associated with a certain factor set α. It tells us also under which condition $|\mathrm{IrrProj}(G, \alpha)|$ is strictly less then $|\mathrm{Irr}(G)|$.

Proposition 2.5 (see [20])   *Let* $R = M(G).G$ *be a representation group of a finite group* G, *where* M(G) *denotes the Schur multiplier of* G. *Then the number of irreducible characters* $\mathrm{Irr}(R)$ *of* R *which lies over a linear character* θ *of* M(G) *is less or equal to* $|\mathrm{Irr}(G)|$.

Proof — The number of irreducible characters $\mathrm{Irr}(R)$ of R which lies over a linear character $\theta \in \mathrm{Irr}M(G)$ is given by

$$\sum_{\chi \in \mathrm{Irr}(R)} \frac{\langle \chi \downarrow_{M(G)}, \theta \rangle}{\chi(1)}.$$

It is known that the quantity

$$\sum_{\chi \in \mathrm{Irr}(R)} \frac{\chi(x)}{\chi(1)} \geqslant 0$$

for each $x \in M(G)$, and it is non-zero if x is a commutator in R. For any $\theta \in \mathrm{Irr}(M(G))$, we have

$$\sum_{x \in M(G)} \sum_{\chi \in \mathrm{Irr}(R)} \frac{\chi(x)\theta(x^{-1})}{\chi(1)} \leqslant \sum_{x \in M(G)} \sum_{\chi \in \mathrm{Irr}(R)} \frac{\chi(x)}{\chi(1)} = |M(G)| |[g]_{R/M(G)}|,$$

where $|[g]_{R/M(G)}|$ is the number of conjugacy classes of $R/M(G) \simeq G$. The last equality follows because the irreducible characters of R with $M(G)$ in their kernels are precisely those which contain the trivial character on the restriction to $M(G)$. Hence

$$\frac{1}{|M(G)|} \sum_{x \in M(G)} \sum_{\chi \in Irr(R)} \frac{\chi(x)\theta(x^{-1})}{\chi(1)}$$

$$= \sum_{\chi \in Irr(R)} \frac{1}{|M(G)|} \sum_{x \in M(G)} \frac{\chi(x)\theta(x^{-1})}{\chi(1)}$$

$$= \sum_{\chi \in Irr(R)} \frac{\langle \chi \downarrow_{M(G)}, \theta \rangle}{\chi(1)} \leqslant |[g]_{R/M(G)}| = |Irr(G)|.$$

Furthermore, if there is a non-identity element $x \in M(G) \setminus \ker(\theta)$ which is a commutator in R, then the inequality becomes strict. □

3 Theory of Fischer-Clifford matrices

Since the character table of $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$ will be constructed by the technique of Fischer-Clifford matrices, we will give a summary of this technique as found in [1].

Let $\overline{G} = N{\cdot}G$ be an extension of N by G, where N is normal in $\overline{G}$ and $\overline{G}/N \simeq G$. Denote the set of all irreducible characters of a finite group G_1 by $Irr(G_1)$. Also, define

$$\overline{H} = \left\{ x \in \overline{G} \mid \theta^x = \theta \right\} = I_{\overline{G}}(\theta)$$

as the inertia group of $\theta \in Irr(N)$ in $\overline{G}$ then N is normal in $\overline{H}$. Let $\overline{g} \in \overline{G}$ be a lifting of $g \in G$ under the natural homomorphism $\overline{G} \longrightarrow G$ and $[g]$ be a conjugacy class of elements with representative g. Let

$$X(g) = \{x_1, x_2, \ldots, x_{c(g)}\}$$

be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \overline{g}$. Now let $\theta_1 = 1_N, \theta_2, \ldots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$ such that for $1 \leqslant i \leqslant t$, we

have $\overline{H_i}$ with corresponding inertia factors H_i. By Gallagher [10] we obtain

$$\mathrm{Irr}(\overline{G}) = \bigcup_{i=1}^{t} \{(\psi_i \overline{\beta})^{\overline{G}} \mid \beta \in \mathrm{IrrProj}(H_i), \text{ with factor set } \alpha_i^{-1}\},$$

where ψ_i is a projective character of $\overline{H_i}$ with factor set $\overline{\alpha}_i$ such that

$$\psi_i \downarrow_N = \theta_i.$$

Observe that α_i and $\overline{\beta}$ are obtained from $\overline{\alpha_i}$ and β, respectively. We have that $\overline{H_1} = \overline{G}$ and $H_1 = G$. Choose $y_1, y_2, \ldots, y_r$ to be representatives of the α_i^{-1}-conjugacy classes of elements of H_i that fuse to $[g]$ in G. We define

$$R(g) = \{(i, y_k) \mid 1 \leqslant i \leqslant t, H_i \cap [g] \neq \emptyset, 1 \leqslant k \leqslant r\}$$

and we note that y_k runs over representatives of the α_i^{-1}-conjugacy classes of elements of H_i which fuse into $[g]$ in G. We define $y_{l_k} \in \overline{H_i}$ such that y_{l_k} ranges over all representatives of the conjugacy classes of elements of $\overline{H_i}$ which map to y_k under the homomorphism

$$\overline{H_i} \longrightarrow H_i$$

whose kernel is N. Then we define the Fischer-Clifford matrix by

$$M(g) = \left(a^j_{(i, y_k)} \right),$$

where

$$a^j_{(i, y_k)} = \sum_{l}{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|} \psi_i(y_{l_k}),$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_{l}'$ is the summation over all l for which $y_{l_k} \sim x_j$ in $\overline{G}$. We also write the Fischer-Clifford matrix for the class $[g]$ as

$$M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}.$$

where, if $H_i \cap [g] = \emptyset$, then the submatrix $M_i(g)$ (corresponding to the inertia group $\overline{H_i}$ and its inertia factor H_i) is not defined and is omitted from $M(g)$. $M(g)$ is a $l \times c(g)$ matrix, where l is the number of α_i^{-1}- regular conjugacy classes of the inertia factors H_i's, $1 \leqslant i \leqslant t$, which fuse into $[g]$ in G and $c(g)$ is the number of conjugacy classes of $\overline{G}$ which correspond to the coset $N\overline{g}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \ldots, x_{c(g)}\}$ is given by

$$
\begin{bmatrix}
C_1(g) \, M_1(g) \\
C_2(g) \, M_2(g) \\
\vdots \\
C_t(g) \, M_t(g)
\end{bmatrix}
$$

where the Fischer-Clifford matrix $M(g)$ is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H_i}$ and $C_i(g)$ is the partial character table of H_i with factor set α_i^{-1} consisting of the columns corresponding to the α_i^{-1}-regular classes that fuse into $[g]$ in G. We obtain the characters of $\overline{G}$ by multiplying the relevant columns of the projective characters of H_i with factor set α_i^{-1} by the rows of $M(g)$. We can also observe that

$$
|\mathrm{Irr}(\overline{G})| = \sum_{i=1}^{t} |\mathrm{IrrProj}(H_i, \alpha_i^{-1})|.
$$

4   The group $2^{4+6}{:}(A_5 \times 3)$

The group
$$
\overline{G} = N_{G_2(4)}((2A)^4) = 2^{4+6}{:}(A_5 \times 3)
$$

is a split extension of the special 2-group 2^{4+6} by $A_5 \times 3$ and is constructed within MAGMA using a permutation representation of degree 416 obtained from Wilson's online ATLAS of Group Representations [24]. The center $Z(2^{4+6}) = 2^4$ and the quotient

$$
2^{4+6}/2^4 \simeq 2^6
$$

are elementary abelian 2-groups and it is also confirmed with the aid of MAGMA that 2^6 is a normal subgroup of $\overline{G}$ and hence

$$2^{4+6}{:}(A_5 \times 3)$$

can be considered as an extension group of 2^6 by $2^4{:}(A_5 \times 3)$. Moreover, with the use of appropriate MAGMA commands it is found that

$$2^{4+6}{:}(A_5 \times 3)$$

is isomorphic to a non-split extension $2^{6\cdot}(2^4{:}(A_5 \times 3))$. Also, using the MAGMA commands "M:= GModule($\overline{G}$,2^6)" and "M:Maximal" the group $\overline{G}/2^6 \simeq 2^4{:}(A_5 \times 3)$ is represented as a matrix group of dimension 6 over GF(2). The generators g_1 and g_2 of $2^4{:}(A_5 \times 3)$, with respective orders of 2 and 15, are as follows:

$$g_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

5 The action of $2^4{:}(A_5 \times 3)$ on 2^6 and $\mathrm{Irr}(2^6)$

For the remainder of the paper we represent $\overline{G}$ by its non-split form, that is

$$\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$$

a non-split extension of $N = 2^6$ by

$$G = 2^4{:}(A_5 \times 3),$$

where N is the vector space of dimension 6 over GF(2) on which the linear group $G = \langle g_1, g_2 \rangle$ acts. When G acts on the conjugacy classes of elements of 2^6, we obtain 3 orbits of lengths $1, 15$ and 48 with respective point stabilizers $P_1 = G$, $P_2 = 2^4{:}(2^2{:}3)$ and $P_3 = A_5$. Since G has 3 orbits on N, then by Brauer's Theorem [7] the action of G on $\mathrm{Irr}(N)$ will also has 3 orbits. The lengths of these orbits are $1, 3$ and 60, with corresponding inertia factor groups

$$H_1 = G, \quad H_2 = 2^4{:}A_5 \quad \text{and} \quad H_3 = 2^2{:}A_4.$$

Note that H_2 is isomorphic to the Mathieu group M_{20} (see [24]). See Table 5.1 for a summary of the action of $2^4{:}(A_5 \times 3)$ on 2^6 and $\mathrm{Irr}(2^6)$, respectively.

Table 5.1: Action of G on N and Irr(N)

	Action of G on N	Action of G on Irr(N)
Number and size of Orbits O_i	$\|O_1\| = 1$ $\|O_2\| = 15$ $\|O_3\| = 48$	$\|O_1\| = 1$ $\|O_2\| = 3$ $\|O_3\| = 60$
Structure of point stabilizers P_i and H_i	$P_1 = 2^4{:}(A_5 \times 3)$ $P_2 = 2^4{:}(2^2{:}3)$ $P_3 = A_5$	$H_1 = 2^4{:}(A_5 \times 3)$ $H_2 = 2^4{:}A_5$ $H_3 = 2^2{:}A_4$
Size of stabilizers P_i and H_i	$\|P_1\| = 2880$ $\|P_2\| = 192$ $\|P_3\| = 60$	$\|H_1\| = 2880$ $\|H_2\| = 960$ $\|H_3\| = 48$
Number of conjugacy classes $[g]$ of P_i and H_i	$\|[g]_{P_1}\| = 19$ $\|[g]_{P_2}\| = 17$ $\|[g]_{P_3}\| = 5$	$\|[g]_{H_1}\| = 19$ $\|[g]_{H_2}\| = 9$ $\|[g]_{H_3}\| = 16$

6 Fusion maps of inertia factors into $2^4{:}(A_5 \times 3)$

We obtain the fusion maps (Tables 6.1 and 6.2) of the inertia factors H_2 and H_3 into G by using their permutation characters in G of degree 3 and 60, respectively and if necessary direct computation in MAGMA.

Table 6.1: The fusion of H_2 into G

$[h]_{H_2}$	$\longrightarrow$	$[g]_G$	$[h]_{H_2}$	$\longrightarrow$	$[g]_G$
1A		1A	4B		4A
2A		2A	4C		4A
2B		2B	5A		5A
3A		3E	5B		5B
4A		4A			

Table 6.2: The fusion of H_3 into G

$[h]_{H_3}$	$\longrightarrow$	$[g]_G$	$[h]_{H_3}$	$\longrightarrow$	$[g]_G$	$[h]_{H_3}$	$\longrightarrow$	$[g]_G$
1A		1A	2F		2B	6B		6D
2A		2B	2G		2B	6C		6C
2B		2B	3A		3A	6D		6D
2C		2B	3B		3B	6E		6C
2D		2A	6A		6C	6F		6D
2E		2B						

7 Projective character tables of inertia factors

In this section, it will be shown that a projective character table with a non-trivial factor set for H_2 and the ordinary character table for H_3 are required in the construction of the character table of $\overline{G}$. Readers are referred to [1],[14] and [15] for the computational techniques being used in this section.

The Fischer-Clifford matrix $M(1A)$ is obtained by using the properties of Fischer-Clifford matrices (see[1]). The action of G on N results in 3 orbits being formed and hence we obtained 3 classes 1A, 2A and 2B of elements of $\overline{G}$ with centralizer orders of 184320, 12288 and 3840, respectively. The matrix $M(1A)$ with corresponding weights attached to rows and columns is given as:

$$M(1A) = \begin{array}{c} \\ 2880 \\ 960 \\ 48 \\ \\ \end{array} \begin{array}{ccc} 184320 & 12288 & 3840 \\ \left(\begin{array}{ccc} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 60 & -4 & 0 \\ 1 & 15 & 48 \end{array}\right) \end{array}$$

Let consider $65a, 78a \in \mathrm{Irr}(G_2(4))$ as listed in the ATLAS [4]. Then we have

$[x]_{G_2(4)}$	1A	2A	2B
65a	65	1	5
78a	78	14	-6

Let γ_1, γ_2 and γ_3 be the rows of the Fischer-Clifford matrix $M(1A)$. Since $\langle (65a)_N, 1_N \rangle = 5$ and $\langle (78a)_N, 1_N \rangle = 0$ we have the following decompositions $(65a)_N = 5\gamma_1 + \gamma_3$ and $(78a)_N = 6\gamma_2 + \gamma_3$. Now by considering the coefficient of γ_3 we deduce that we have a character $\chi \in \mathrm{Irr}(\overline{G})$ with $\deg(\chi) = 60$. If

$$[x_1, x_2, x_3, \ldots, x_s]$$

is the transpose of the partial entries for the projective characters of H_3 on $1A$, then

$$C_3(1A)M_3(1A)$$

is a $s \times 3$ matrix with the first entry $60x_1 = 60$. Here $C_3(1A)$ is the partial character table (ordinary or projective) of H_3 corresponding to the identity class of H_3 and $M_3(1A)$ is the third row of entries of $M(1A)$ associated with the inertia factor group H_3. Hence $x_1 = 1$ and this shows that the character table of H_3 that will be used contains a character of degree 1. Thus the partial character table $C_3(1A)$ comes from the ordinary characters of H_3.

Similarly, by considering the coefficients of γ_2 in the decomposition of $(78a)_N$, we obtain that there is an irreducible character of $\overline{G}$ of degree 18. Let

$$[y_1, y_2, y_3, \ldots, y_t]$$

be the transpose of the partial entries for the projective character table of H_2 on $1A$. Then $C_2(1A)M_2(1A)$ is a $t \times 3$ matrix with the first entry $3y_1 = 18$ and hence we obtain that $y_1 = 6$. This shows that the partial projective character table $C_2(1A)$ of H_2 should contain a character of degree 6. But the ordinary character table of H_2 does not contain a character of degree 6 and therefore a projective character table $\mathrm{IrrProj}(H_2, \alpha^{-1})$ of H_2, with a non-trivial factor set α^{-1}, will be used in the construction of the character table of $\overline{G}$.

Since the Schur multiplier of $M(H_2) = \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ is the abelian group of order 32, we have 7 classes of order 2 and 24 classes of order 4. Hence we have 7 projective characters tables with factor sets of order 2 and 24 projective character tables with factor sets of order 4. We know that the degrees of the characters of the desire projective character table $\mathrm{IrrProj}(H_2, \alpha^{-1})$ of H_2 must be divisible by the order of the corresponding factor set α^{-1}. Now a non-trivial class of $M(H_2)$ has either order of 2 or 4 and we also showed earlier that the set $\mathrm{IrrProj}(H_2, \alpha^{-1})$ contains an irreducible projective character of degree 6. But 4 does not divide 6, hence we deduce that

a set $\mathrm{IrrProj}(H_2, \alpha^{-1})$ such that $\alpha^2 \sim 1$ is needed to construct the set $\mathrm{Irr}(\overline{G})$.

Haggarty and Humphreys [8] show that it is possible to determine the projective characters of H_2 with a given factor set α^{-1}, without the full representation group $M(H_2){\cdot}H_2$ of H_2. So the aim is to find a double cover of H_2 which contains the correct choice of $\mathrm{IrrProj}(H_2, \alpha^{-1})$. Since we have a known permutation representation of $\overline{G}$, the inertia group

$$\overline{H_2} = 2^{6\cdot}H_2$$

of $\overline{G}$ on $\mathrm{Irr}(N)$ is generated within GAP. Next, we compute the normal subgroups of $\overline{H_2}$ which are contained in N and found there is a normal subgroup $N_1 = 2^5$ of order 32 in $\overline{H_2}$. The factor group

$$R = \overline{H_2}/N_1$$

is the double cover $2.H_2$ which we are interested in. Thus the desired projective characters of H_2 with factor set α^{-1} such that $\alpha^2 \sim 1$ can be determined from the ordinary character table of the double cover R. The ordinary character table of R is computed and it is found that $|\mathrm{Irr}(R)| = 16$, where 9 of these are the liftings of the ordinary irreducible characters of H_2 to R, while the other 7 ordinary characters represent the desired set $\mathrm{IrrProj}(H_2, \alpha^{-1})$ needed in the construction of the set $\mathrm{Irr}(\overline{G})$. The following GAP code was use to "extract" the set $\mathrm{IrrProj}(H_2, \alpha^{-1})$ (see Table 7.1) from the set $\mathrm{Irr}(R)$:

```
gap> t:= CharacterTable("H₂");
gap> 2t:= CharacterTable("R");
gap> F:=GetFusionMap( 2t, t );
gap> map:= ProjectionMap(F);
gap> projchars:=List(2t,x− >x{map});
```

Hence we can formulate the following theorem with $\overline{G}, N, N_1, G, H_2$, H_3 and $\overline{H_2}$ defined as above.

Theorem 7.1 *The sets*

$$\mathrm{Irr}(G), \quad \mathrm{Irr}(H_3) \quad \text{and} \quad \mathrm{IrrProj}(H_2, \alpha^{-1})$$

such that $\alpha^2 \sim 1$ are required in the construction of the set $\mathrm{Irr}(\overline{G})$ using Fischer-Clifford matrices. Moreover, $\mathrm{IrrProj}(H_2, \alpha^{-1})$ is obtained

from $\underline{\mathrm{Irr}}(2.H_2)$, *where the double cover* $2.H_2$ *is isomorphic to the quotient* $\overline{H_2}/N_1$.

Table 7.1: Projective character table of H_2 with factor set α^{-1}

$[h]_{H_2}$	1A	2A	2B	3A	4A	4B	4C	5A	5B
$|C_{H_2}(h)|$	960	64	16	3	16	16	16	5	5
ϕ_1	6	-2	-2	0	0	0	2	1	1
ϕ_2	6	-2	2	0	0	0	-2	1	1
ϕ_3	10	2	2	1	0	0	2	0	0
ϕ_4	10	2	-2	1	0	0	-2	0	0
ϕ_5	12	-4	0	0	0	0	0	A	A*
ϕ_6	12	-4	0	0	0	0	0	A*	A
ϕ_7	20	4	0	-1	0	0	0	0	0

where $A = \frac{1}{2}(-1+\sqrt{5})$

8 Fischer-Clifford Matrices of $\overline{G}$

The fusion of classes $[h]_{H_i}$ of elements of the inertia factors H_i into a class $[g]$ of G will determine the size of a Fischer-Clifford matrix $M(g)$. The sizes of these matrices $M(g)$ vary from 1×1 to 8×8 matrices. Note that only the fusion of the α_i^{-1}- regular classes of H_2 into the classes of G is permitted. Hence, we have the number of conjugacy classes of $\overline{G}$ lying above a class $[g]$ of G and the centralizer orders of these classes are given by the equation

$$|C_{\overline{G}}(x)| = \frac{k|C_G(g)|}{f_j}$$

which is obtained from the method of coset analysis. The values of the k's can be obtained by evaluating the permutation character

$$\chi(G|N) = 1aaabc + 4a + 5aa + 15abc$$

of G on N for each class representative $g \in G$ (see, for example, [13],[17] and [18]). The properties of Fischer-Clifford matri-

ces (see [1]), the equation for the centralizer order $|C_{\overline{G}}(x)|$, the permutation character

$$\chi(G_2(4)|\overline{G}) = 1a + 350a + 364b + 650a$$

of $G_2(4)$ on the classes of $\overline{G}$ and some computational techniques used in [14] are used to compute the conjugacy classes (see Table 8.2) and entries of the Fischer-Clifford matrices (see Table 8.1) of $\overline{G}$ which correspond to a coset $N\overline{g}$, $g \in G$. Also, we made use of the information about the classes of $\overline{G}$ which can be obtained from the uploaded character table of $2^{4+6}{:}(A_5 \times 3)$ in GAP. The fusion map of $\overline{G}$ into $G_2(4)$ is also found in the last column of Table 8.2.

For example, consider the conjugacy class 2A of G. Observe that the only class fusions into 2A are from the α^{-1}-regular class 2A of H_2 and the class 2D of H_3. Hence the Fischer-Clifford matrix M(2A) will be a 3×3 matrix. Therefore the coset $N\overline{g}$, for a class representative g in 2A, is splitting into 3 classes $[x_1]_{\overline{G}}$, $[x_2]_{\overline{G}}$ and $[x_3]_{\overline{G}}$ of $\overline{G}$. Then we obtain that M(2A) has the following form with corresponding weights attached to the rows and columns:

$$M(2A) = \begin{array}{c} |C_{H_1}(2A)| \\ |C_{H_2}(2A)| \\ |C_{H_3}(2D)| \end{array} \overset{\displaystyle |C_{\overline{G}}(x_1)| \quad |C_{\overline{G}}(x_2)| \quad |C_{\overline{G}}(x_3)|}{\left(\begin{array}{ccc} a & d & g \\ b & e & h \\ c & f & i \end{array} \right)}.$$

By Theorem 1.3 found in [1], we have the following form of M(2A):

$$M(2A) = \begin{array}{c} 192 \\ 64 \\ 16 \end{array} \overset{\displaystyle |C_{\overline{G}}(x_1)| \quad |C_{\overline{G}}(x_2)| \quad |C_{\overline{G}}(x_3)|}{\left(\begin{array}{ccc} 1 & 1 & 1 \\ b & e & h \\ c & f & i \end{array} \right)}.$$

Further, if we assume that $N\overline{g}$ is a split coset (see Definition 2.5 in [1]), then the first property of Lemma 2.8 (found in [1]) applies and M(2A) will have the form:

$$M(2A) = \begin{array}{c} 192 \\ 64 \\ 16 \end{array} \overset{\displaystyle |C_{\overline{G}}(x_1)| \quad |C_{\overline{G}}(x_2)| \quad |C_{\overline{G}}(x_3)|}{\left(\begin{array}{ccc} 1 & 1 & 1 \\ 3 & e & h \\ 12 & f & i \end{array} \right)}.$$

Then by the column orthogonality property (c) in [1], it follows that

$$|C_{\overline{G}}(x_1)| = 192 + 64\text{x}9 + 16\text{x}144 = 3072.$$

But according to the uploaded character table of $2^{4+6}{:}(A_5 \times 3)$ in GAP, there does not exist an element $y \in \overline{G}$ such that the centralizer order

$$|C_{\overline{G}}(y)| = 3072.$$

Hence $N\overline{g}$ is not a split coset and $M(2A)$ cannot assume the above form. If the two classes of involutions 2A and 2B of $\overline{G}$ coming from the identity coset N are excluded, then it follows from the uploaded character table of $2^{4+6}{:}(A_5 \times 3)$ in GAP that the remaining 11 classes of order two and four of $\overline{G}$ must come from the cosets corresponding to the classes 2A, 2B and 4A of G. But again according to the up-loaded character table of $2^{4+6}{:}(A_5 \times 3)$, there are only two classes, 8A and 8B, of order eight for $\overline{G}$. Since G does not have any classes of order eight, we must have that the classes 8A and 8B of $\overline{G}$ must come from the class 4A of G. Therefore the 11 classes $[x_j]_{\overline{G}}$ of order 2 and 4 of $\overline{G}$ must be obtained from the classes 2A and 2B of G. Since the coset $N\overline{g}$ which is obtained from the class 2B of G is a split coset, the centralizer orders $|C_{\overline{G}}(x_j)|$ of the eight classes $[x_j]$ coming from this coset (see Table 8.2) are easily computed from the corresponding Fischer-Clifford matrix $M(2B)$ using the matrix properties in [1]. The centralizer orders 768, 512 and 512 of the remaining 3 classes of order 2 and 4 of $\overline{G}$ obtained from the uploaded character table of $2^{4+6}{:}(A_5 \times 3)$ must come from the coset associated with the class 2A of G. Together with the remaining properties of Fischer-Clifford matrices found in [1], we conclude that $M(2A)$ must have the following form

$$M(2A) = \begin{matrix} & & 768 & 512 & 512 \\ & 192 \\ & 64 \\ & 16 \end{matrix} \left(\begin{matrix} 1 & 1 & 1 \\ 3 & -1 & -1 \\ 0 & 4 & -4 \end{matrix} \right).$$

The other Fischer-Clifford matrices were obtained in a much easier way using the properties in [1].

Table 8.1: The Fischer-Clifford Matrices of $\overline{G}$

$M(g)$	$M(g)$

$$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & -1 \\ 60 & -4 & 0 \end{pmatrix}$$

$$M(2A) = \begin{pmatrix} 1 & 1 & 1 \\ 3 & -1 & -1 \\ 0 & 4 & -4 \end{pmatrix}$$

$$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -3 & 3 & 3 & -3 & 1 & -1 & -1 & 1 \\ -3 & 3 & -3 & 3 & 1 & -1 & 1 & -1 \\ 3 & 3 & -3 & -3 & -1 & -1 & 1 & 1 \end{pmatrix}$$

$$M(3A) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$$

$$M(3B) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$$

$$M(3C) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(3D) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(3E) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

$$M(4A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$M(5A) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

$$M(5B) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

$$M(6A) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(6B) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(6C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

$$M(6D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

$$M(15A) = \begin{pmatrix} 1 \end{pmatrix}$$
$$M(15C) = \begin{pmatrix} 1 \end{pmatrix}$$

$$M(15B) = \begin{pmatrix} 1 \end{pmatrix}$$
$$M(15D) = \begin{pmatrix} 1 \end{pmatrix}$$

Table 8.2: The classes of $\overline{G}$

$[g]_G$	k	f_j	$[x]_{\overline{G}}$	$|C_{\overline{G}}(x)|$	$\to [y]_{G_2(4)}$
1A	64	$f_1 = 1$	1A	184320	1A
		$f_2 = 15$	2A	12288	2A
		$f_3 = 48$	2B	3840	2B
2A	16	$f_1 = 4$	2C	768	2B
		$f_2 = 6$	4A	512	4A
		$f_3 = 6$	4B	512	4C
2B	16	$f_1 = 1$	2D	768	2A
		$f_2 = 1$	4C	768	4A
		$f_3 = 1$	4D	768	4B
		$f_4 = 1$	4E	768	4B
		$f_5 = 3$	2E	256	2B
		$f_6 = 3$	4F	256	4B
		$f_7 = 3$	4G	256	4C
		$f_8 = 3$	4H	256	4C
3A	16	$f_1 = 1$	3A	2880	3A
		$f_2 = 15$	6A	192	6A
3B	16	$f_1 = 1$	3B	2880	3A
		$f_2 = 15$	6B	192	6A
3C	1	$f_1 = 1$	3C	36	3B
3D	1	$f_1 = 1$	3D	36	3B
3E	4	$f_1 = 1$	3E	36	3B
		$f_2 = 3$	6C	12	6B
4A	4	$f_1 = 2$	8A	32	8A
		$f_2 = 2$	8B	32	8B
5A	4	$f_1 = 1$	5A	60	5A
		$f_2 = 3$	10A	20	10C
5B	4	$f_1 = 1$	5B	60	5B
		$f_2 = 3$	10B	20	10D
6A	1	$f_1 = 1$	6D	12	6B
6B	1	$f_1 = 1$	6E	12	6B
6C	4	$f_1 = 1$	6F	48	6A
		$f_2 = 1$	12A	48	12A
		$f_3 = 1$	12B	48	12B
		$f_4 = 1$	12C	48	12C
6D	4	$f_1 = 1$	6G	48	6A
		$f_2 = 1$	12D	48	12A
		$f_3 = 1$	12E	48	12C
		$f_4 = 1$	12F	48	12B
15A	1	$f_1 = 1$	15A	15	15A
15B	1	$f_1 = 1$	15B	15	15A
15C	1	$f_1 = 1$	15C	15	15B
15D	1	$f_1 = 1$	15D	15	15B

9 The character table of $\overline{G}$

Having obtained the Fischer-Clifford matrices of $\overline{G}$, the projective characters of the inertia factor H_2, the ordinary characters of H_1 and H_3, the fusion of the α_i^{-1}- regular classes of H_2 into G, and the fusion of the classes of H_3 into G, we are able to construct the ordinary character table of $\overline{G}$ using Fischer-Clifford theory. The set of irreducible characters of $\overline{G}$ will be partitioned into 3 blocks, namely $\triangle_1 = \{\chi_j \mid 1 \leqslant j \leqslant 19\}$, $\triangle_2 = \{\chi_j \mid 20 \leqslant j \leqslant 26\}$ and $\triangle_3 = \{\chi_j \mid 27 \leqslant j \leqslant 42\}$ corresponding to the inertia factor groups H_1, H_2 and H_3, respectively, where $\chi_j \in \mathrm{Irr}(\overline{G})$. The consistency and accuracy of the character table of $\overline{G}$ (see Table 9.1) have been tested by using the GAP code labeled as Programme E in [23].

Table 9.1: The character table of $\overline{G} = 2^{6\cdot}(2^4{:}(A_5 \times 3))$

$[g]_G$	1A			2A			2B								3A		3B		3C	3D	3E	
$[g]_{\overline{G}}$	1A	2A	2B	2C	4A	4B	2D	4C	4D	4E	2E	4F	4G	4H	3A	6A	3B	6B	3C	3D	3E	6E
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	A	A	$\overline{A}$	$\overline{A}$	A	$\overline{A}$	1	1
χ_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\overline{A}$	$\overline{A}$	A	A	$\overline{A}$	A	1	1
χ_4	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	0	0	0	0
χ_5	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	0	0	0	0
χ_6	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	B	B	$\overline{B}$	$\overline{B}$	0	0	0	0
χ_7	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	B	B	$\overline{B}$	$\overline{B}$	0	0	0	0
χ_8	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	$\overline{B}$	$\overline{B}$	B	B	0	0	0	0
χ_9	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	$\overline{B}$	$\overline{B}$	B	B	0	0	0	0
χ_{10}	4	4	4	4	4	4	0	0	0	0	0	0	0	0	4	4	4	4	1	1	1	1
χ_{11}	4	4	4	4	4	4	0	0	0	0	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$	A	$\overline{A}$	1	1
χ_{12}	4	4	4	4	4	4	0	0	0	0	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C	$\overline{A}$	A	1	1
χ_{13}	5	5	5	5	5	5	1	1	1	1	1	1	1	1	5	5	5	5	-1	-1	-1	-1
χ_{14}	5	5	5	5	5	5	1	1	1	1	1	1	1	1	D	D	$\overline{D}$	$\overline{D}$	-A	-$\overline{A}$	-1	-1
χ_{15}	5	5	5	5	5	5	1	1	1	1	1	1	1	1	$\overline{D}$	$\overline{D}$	D	D	-$\overline{A}$	-A	-1	-1
χ_{16}	15	15	15	-1	-1	-1	3	3	3	3	3	3	3	3	0	0	0	0	3	3	0	0
χ_{17}	15	15	15	-1	-1	-1	3	3	3	3	3	3	3	3	0	0	0	0	B	$\overline{B}$	0	0
χ_{18}	15	15	15	-1	-1	-1	3	3	3	3	3	3	3	3	0	0	0	0	$\overline{B}$	B	0	0
χ_{19}	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0	0	0

χ_{20}	18	18	-6	-6	2	2	-6	-6	-6	-6	2	2	2	2	0	0	0	0	0	0	0	0
χ_{21}	18	18	-6	-6	2	2	6	6	6	6	-2	-2	-2	-2	0	0	0	0	0	0	0	0
χ_{22}	30	30	-10	6	-2	-2	6	6	6	6	-2	-2	-2	-2	0	0	0	0	0	0	3	-1
χ_{23}	30	30	-10	6	-2	-2	-6	-6	-6	-6	2	2	2	2	0	0	0	0	0	0	3	-1
χ_{24}	36	36	-12	-12	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	36	36	-12	-12	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}	60	60	-20	12	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3	1
χ_{27}	60	-4	0	0	4	-4	0	8	-4	-4	4	-4	0	0	15	-1	15	-1	0	0	0	0
χ_{28}	60	-4	0	0	4	-4	-4	-4	0	8	0	0	4	-4	15	-1	15	-1	0	0	0	0
χ_{29}	60	-4	0	0	4	-4	-4	-4	8	0	0	0	-4	4	15	-1	15	-1	0	0	0	0
χ_{30}	60	-4	0	0	4	-4	8	0	-4	-4	-4	4	0	0	15	-1	15	-1	0	0	0	0
χ_{31}	60	-4	0	0	4	-4	-4	-4	0	8	0	0	4	-4	E	$-\overline{A}$	$\overline{E}$	$-A$	0	0	0	0
χ_{32}	60	-4	0	0	4	-4	-4	-4	0	8	0	0	4	-4	$\overline{E}$	$-A$	E	$-\overline{A}$	0	0	0	0
χ_{33}	60	-4	0	0	4	-4	-4	-4	8	0	0	0	-4	4	E	$-\overline{A}$	$\overline{E}$	$-A$	0	0	0	0
χ_{34}	60	-4	0	0	4	-4	-4	-4	8	0	0	0	-4	4	$\overline{E}$	$-A$	E	$-\overline{A}$	0	0	0	0
χ_{35}	60	-4	0	0	4	-4	8	0	-4	-4	-4	4	0	0	E	$-\overline{A}$	$\overline{E}$	$-A$	0	0	0	0
χ_{36}	60	-4	0	0	4	-4	8	0	-4	-4	-4	4	0	0	$\overline{E}$	$-A$	E	$-\overline{A}$	0	0	0	0
χ_{37}	60	-4	0	0	4	-4	0	8	-4	-4	4	-4	0	0	E	$-\overline{A}$	$\overline{E}$	$-A$	0	0	0	0
χ_{38}	60	-4	0	0	4	-4	0	8	-4	-4	4	-4	0	0	$\overline{E}$	$-A$	E	$-\overline{A}$	0	0	0	0
χ_{39}	180	-12	0	0	-4	4	0	0	12	-12	-4	-4	8	0	0	0	0	0	0	0	0	0
χ_{40}	180	-12	0	0	-4	4	0	0	-12	12	-4	-4	0	8	0	0	0	0	0	0	0	0
χ_{41}	180	-12	0	0	-4	4	-12	12	0	0	0	8	-4	-4	0	0	0	0	0	0	0	0
χ_{42}	180	-12	0	0	-4	4	12	-12	0	0	8	0	-4	-4	0	0	0	0	0	0	0	0

where $A = \frac{-1-\sqrt{3}i}{2}$, $B = \frac{-3-3\sqrt{3}i}{2}$, $C = -2 - 2\sqrt{3}i$, $D = \frac{-5-5\sqrt{3}i}{2}$, $E = \frac{-15+15\sqrt{3}i}{2}$.

Table 9.1 (continued)

$[g]_G$	4A		5A		5B		6A	6B	6C				6D				15A	15B	15C	15D
$[g]_{\overline{G}}$	8A	8B	5A	10A	5B	10B	6D	6E	6F	12A	12B	12C	6G	12D	12E	12F	15A	15B	15C	15D
X_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X_2	1	1	1	1	1	1	A	Ā	A	A	A	A	Ā	Ā	Ā	Ā	A	Ā	A	Ā
X_3	1	1	1	1	1	1	Ā	A	Ā	Ā	Ā	Ā	A	A	A	A	Ā	A	Ā	A
X_4	-1	-1	F	F	*F	*F	0	0	-1	-1	-1	-1	-1	-1	-1	-1	F	F	*F	*F
X_5	-1	-1	*F	*F	F	F	0	0	-1	-1	-1	-1	-1	-1	-1	-1	*F	*F	F	F
X_6	-1	-1	F	F	*F	*F	0	0	-A	-A	-A	-A	-Ā	-Ā	-Ā	-Ā	H	-H̄	I	Ī
X_7	-1	-1	*F	*F	F	F	0	0	-A	-A	-A	-A	-Ā	-Ā	-Ā	-Ā	I	Ī	H	H̄
X_8	-1	-1	F	F	*F	*F	0	0	-Ā	-Ā	-Ā	-Ā	-A	-A	-A	-A	H̄	H	Ī	I
X_9	-1	-1	*F	*F	F	F	0	0	-Ā	-Ā	-Ā	-Ā	-A	-A	-A	-A	Ī	I	H̄	H
X_{10}	0	0	-1	-1	-1	-1	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	-1
X_{11}	0	0	-1	-1	-1	-1	A	Ā	0	0	0	0	0	0	0	0	-A	-Ā	-A	-Ā
X_{12}	0	0	-1	-1	-1	-1	Ā	A	0	0	0	0	0	0	0	0	-Ā	-A	-Ā	-A
X_{13}	1	1	0	0	0	0	-1	-1	1	1	1	1	1	1	1	1	0	0	0	0
X_{14}	1	1	0	0	0	0	-A	-Ā	A	A	A	A	Ā	Ā	Ā	Ā	0	0	0	0
X_{15}	1	1	0	0	0	0	-Ā	-A	Ā	Ā	Ā	Ā	A	A	A	A	0	0	0	0
X_{16}	-1	-1	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
X_{17}	-1	-1	0	0	0	0	-A	-Ā	0	0	0	0	0	0	0	0	0	0	0	0
X_{18}	-1	-1	0	0	0	0	-Ā	-A	0	0	0	0	0	0	0	0	0	0	0	0
X_{19}	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{20}	2	-2	3	-1	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{21}	-2	2	3	-1	3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{22}	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{23}	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{24}	0	0	G	*F	*G	F	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{25}	0	0	*G	F	G	*F	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{26}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{27}	0	0	0	0	0	0	0	0	3	-1	-1	-1	3	-1	-1	-1	0	0	0	0
X_{28}	0	0	0	0	0	0	0	0	-1	-1	-1	3	-1	-1	-1	3	0	0	0	0
X_{29}	0	0	0	0	0	0	0	0	-1	-1	3	-1	-1	-1	3	-1	0	0	0	0
X_{30}	0	0	0	0	0	0	0	0	-1	3	-1	-1	-1	3	-1	-1	0	0	0	0
X_{31}	0	0	0	0	0	0	0	0	-Ā	-Ā	-Ā	B	-A	-A	-A	B	0	0	0	0
X_{32}	0	0	0	0	0	0	0	0	-A	-A	-A	B	-Ā	-Ā	-Ā	B	0	0	0	0
X_{33}	0	0	0	0	0	0	0	0	-Ā	-Ā	B	-Ā	-A	-A	B	-A	0	0	0	0
X_{34}	0	0	0	0	0	0	0	0	-A	-A	B	-A	-Ā	-Ā	B	-Ā	0	0	0	0
X_{35}	0	0	0	0	0	0	0	0	-Ā	B	-Ā	-Ā	-A	B	-A	-A	0	0	0	0
X_{36}	0	0	0	0	0	0	0	0	-A	B	-A	-A	-Ā	B	-Ā	-Ā	0	0	0	0
X_{37}	0	0	0	0	0	0	0	0	B	-Ā	-Ā	-Ā	B	-A	-A	-A	0	0	0	0
X_{38}	0	0	0	0	0	0	0	0	B	-A	-A	-A	B	-Ā	-Ā	-Ā	0	0	0	0
X_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where $F = \frac{1-\sqrt{5}}{2}$, $G = \frac{-3-3\sqrt{5}}{2}$, $H = -2E(15)^7 - E(15)^{13}$,
$I = -E(15) - E(15)^4$.

GAP is used to compute possible power maps from the character table of $\overline{G}$. The GAP code, found in [23] as Programme E, produces unique p-power maps (see Table 9.2) for Table 9.1.

Table 9.2: The power maps of the elements of $2^{6\cdot}(2^4{:}(A_5 \times 3))$

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	$[g]_G$	$[x]_{\overline{G}}$	2	3	5
1A	1A				5B	5B			1A
	2A	1A				10B	5A		2B
	2B	1A							
2A	2C	1A			6A	6D	3D	2C	
	4A	2A							
	4B	2A							
2B	2D	1A			6B	6E	3C	2C	
	4C	2A							
	4D	2A							
	4E	2A							
	2E	1A							
	4F	2A							
	4G	2A							
	4H	2A							
3A	3A		1A		6C	6F	3B	2D	
	6A	3B	2A			12A	6B	4C	
						12B	6B	4E	
						12C	6B	4D	
3B	3B		1A		6D	6G	3A	2D	
	6B	3A	2A			12D	6A	4C	
						12E	6A	4E	
						12F	6A	4D	
3C	3C		1A		15A			5B	3B
3D	3D		1A		15B			5B	3A
3E	3E		1A		15C			5A	3B
	6C	3E	2B						
4A	8A	4A			15D			5A	3A
	8B	4B							
5A	5A			1A					
	10A	5B		2B					

Acknowledgments

REFERENCES

[1] F. ALI – J. MOORI: "The Fischer-Clifford matrices of a maximal subgroup of Fi'_{24}", *Repr. Theory* 7 (2003), 300–321.

[2] A.B.M. BASHEER – J. MOORI: "A survey on Clifford-Fischer theory", in London Math. Soc. Lecture Notes Series 422 (2015), *Cambridge University Press*, 160–172.

[3] W. BOSMA – J.J. CANON: "Handbook of Magma Functions", Department of Mathematics, University of Sydney (November, 1994).

[4] J.H. CONWAY – R.T. CURTIS – S.P. NORTON – R.A. PARKER – R.A. WILSON: "Atlas of Finite Groups", *Oxford University Press*, Oxford (1985).

[5] B. FISCHER: "Clifford-matrices", in Progr. Math. 95 (1991), 1–16.

[6] THE GAP GROUP: "GAP — Groups, Algorithms, and Programming", Version 4.6.3 (2013).

[7] D. GORENSTEIN: "Finite Groups", *Harper and Row*, New York (1968).

[8] R.J. HAGGARTY – J.F. HUMPHREYS, "Projective characters of finite groups", *Proc. London Math. Soc.* 36 (1975), 176–192.

[9] I.M. ISAACS: "Character Theory of Finite Groups", *Academic Press*, San Diego (1976).

[10] G. KARPILOVSKY: "Group Representations: Introduction to Group Representations and Characters", Vol. 1, Part B, *North-Holland*, Amsterdam (1992).

[11] G. KARPILOVSKY: "Projective Representations of Finite Groups", *Marcel Dekker*, New York (1985).

[12] J. MOORI: "On certain groups associated with the smallest Fischer group", *J. London Math. Soc.* 2 (1981), 61–67.

[13] J. MOORI – Z.E. MPONO: "The Fischer-Clifford matrices of the group $2^6{:}SP_6(2)$", *Quaestiones Math.* 22 (1999), 257–298.

[14] A.L. PRINS: "The character table of an involution centralizer in the Dempwolff group $2^5{\cdot}GL_5(2)$", *Quaestiones Math.* 39 (2016), 561–576.

[15] A.L. PRINS: "The projective character tables of a solvable group $2^6{:}(6 \times 2)$", *Int. J. Math. Mathematical Sci.* (2019), 15pp.; doi:10.1155/2019/8684742.

[16] A.L. PRINS: "The number of irreducible projective characters with associated factor set of any finite group"; mathoverflow.net/q/163208 (2014-05-05).

[17] A.L. PRINS – R.L. FRAY: "The Fischer-Clifford matrices of the inertia group $2^7{:}O^-(6,2)$ of a maximal subgroup $2^7{:}SP(6,2)$ in $Sp_8(2)$", *Int. J. Group Theory* 2(3) (2013), 19–38.

[18] A.L. PRINS – R.L. FRAY: "The Fischer-Clifford matrices of an extension group of the form $2^7{:}(2^5{:}S_6)$", *Int. J. Group Theory* 3(2) (2014), 21–39.

[19] E.W. READ: "On the centre of a representation group", *J. London Math. Soc.* 16 (1977), 43–50.

[20] G. ROBINSON: "The number of irreducible projective characters with associated factor set of any finite group"; mathoverflow.net/q/163229 (2014-05-05).

[21] D. ROSSI: "Definition from Character Theory"; math.stackexchange.com/q/2834116 (2018-06-29).

[22] J. SCHMIDT: "Projective characters with corresponding factor set"; mathoverflow.net/q/165226 (2017-04-13).

[23] T.T. SERETLO: "Fischer Clifford Matrices and Character Tables of Certain Groups Associated with Simple Groups $O_{10}^+(2)$, HS and Ly", *PhD Thesis*, University of KwaZulu Natal (2011).

[24] R.A. WILSON – P. WALSH – J. TRIPP – I. SULEIMAN – S. ROGERS – R. PARKER – S. NORTON – S. NICKERSON – S. LINTON – J. BRAY – R. ABBOT: "ATLAS of Finite Group Representations"; brauer.maths.qmul.ac.uk/Atlas/v3/

Abraham Love Prins
Department of Mathematics and Applied Mathematics
Nelson Mandela University
PO Box 77000, Port Elizabeth 6031 (South Africa)
e-mail: abraham.prins@mandela.ac.za